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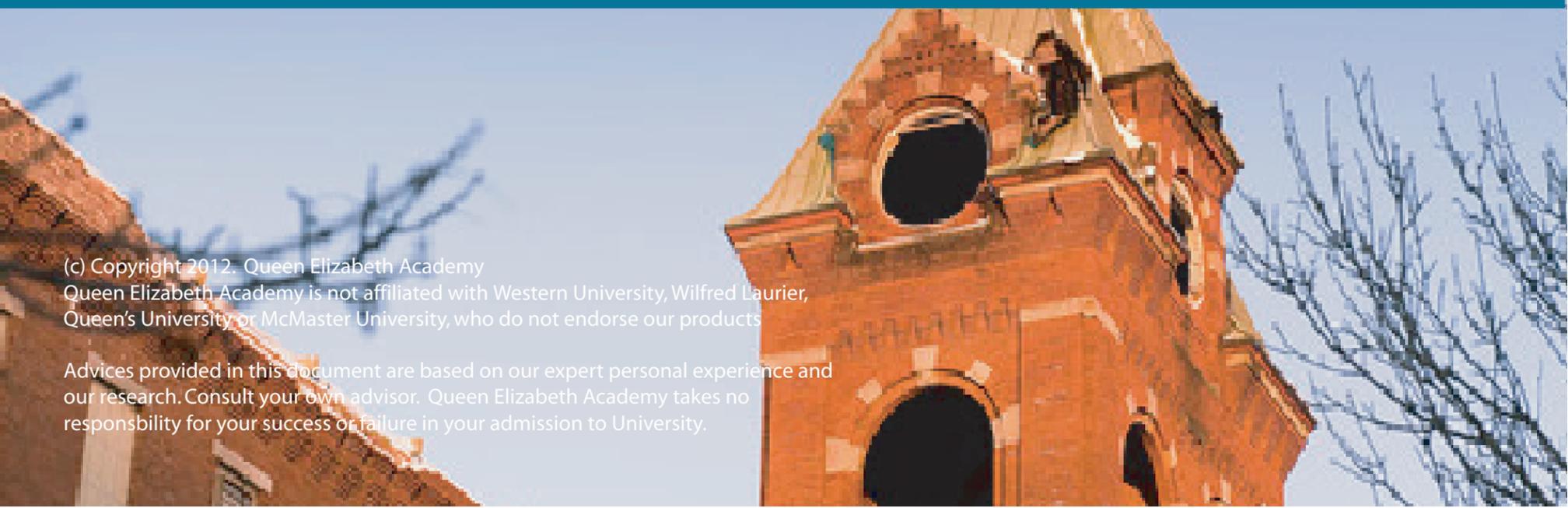
Our Approach to Learning Mathematics

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The troublesome thing with math for a lot of students, is that math knowledge stacks on from previous years. This means that each year builds upon the last, upon the last, upon the last, etc. By the time you hit grade 12, you're going to need a solid understanding of math from grade 8 to grade 11.

Some of the problem lies within the student's own beliefs. Students who perform poorly in math often blame their personal limitation or that they just haven't inherited the right genes for the subject. Wrong. It's all about practicing and building a solid foundation. **You need to keep on top of the skills, maintain all the fundamental skills, and practice, practice, practice!** Just like anything else, you need to put the time in if you want to succeed.

Personal aspects aside, the general root of the problem is this: most people have gaps in their previous years, so when the current year touches on that topic, they're already behind before they've even started.

So how do we tackle this? Well for starters, right away! The longer you wait to catch yourself up, the more you'll fall behind in school. No matter how accommodating school might be, you need to take matters in your own hands to get caught up – time waits for no one.

After working with over 400 students in math, I've seen some of the same problems over and over. However, there is neither substitute nor quick fix for hard work. You need to dedicate a certain amount of time each week to math. The higher the grade, the more time you need to put in – don't forget the concepts get more complex – and hopefully, you can work around some of the concepts.

Here are 4 tips to improve your math marks:

Tip #1: Organizing knowledge

The fact is, for high school level math, there are only so many types of questions they can ask you for each unit. Of course the teacher will probably try and throw in a few curves, but ultimately, the underlying question is going to be the same. If you classify the questions into 4 or 5 types and practice those types regularly, chances are, you'll perform well in the knowledge and application sections.

For example, the first unit in grade 10 is linear systems, where students need to solve for two equations using methods like substitution of elimination.
Most students will practice questions like this:

Type 1

$2x + y = 3$

$x - 3y = 6$

But this isn't enough. In many cases, you'll need to know:

Type 2

$\dfrac{1}{2}x + \dfrac{1}{3}y = 3$

$\dfrac{1}{2}x - \dfrac{1}{4}y = \dfrac{1}{6}$

Type 3

$2(x - 1) + 3(y + 2) = 3$

$3(x + 1) - 4(y - 1) = 6$

Type 4

$\dfrac{(x - 1)}{2} + \dfrac{2(y - 1)}{3} = 4$

$\dfrac{(x + 3)}{4} - \dfrac{3(y - 2)}{5} = 0$

And there are skill items such as *expansion*, *eliminating fraction* and *substituting fraction* that the student needs to beware of, and it is not apparent just by looking at the question, until you actually attempt it.

Now, how will you know how many types of questions there are? This really depends on the unit, so it's more important to scan through your textbook and all your worksheets thoroughly in order to find the different types. Teachers often won't put the straightforward questions on tests, preferring to test students on whether or not they know the variations instead.

We help students do exactly this in our tutoring sessions, teaching the students to classify the types of questions and helping them practice.

Tip #2: Strengthening foundational skills

For most math units in grade 11 and 12, answering questions takes place in steps. The first two steps are usually the new concepts, while the rest of the steps that follow are only used to simplify the solution. If your foundation is weak, you can lose a lot of marks in these simplifying steps (reducing, factoring, simplifying, etc.). It appears as though you have made careless mistakes, but in reality, there are some recurring gaps in your thought process.
If you discover any gaps on the basic skills, then the trick is to focus in on those types of questions. If you can try to aim for 60-80 of those types of questions, then you can be sure

that you've bridged the gap. But where can you find these questions? The library is a great place to start. School websites (your teacher or another) may have some practice questions that you can download. Our tutoring school has a large collection of these practice exercises.
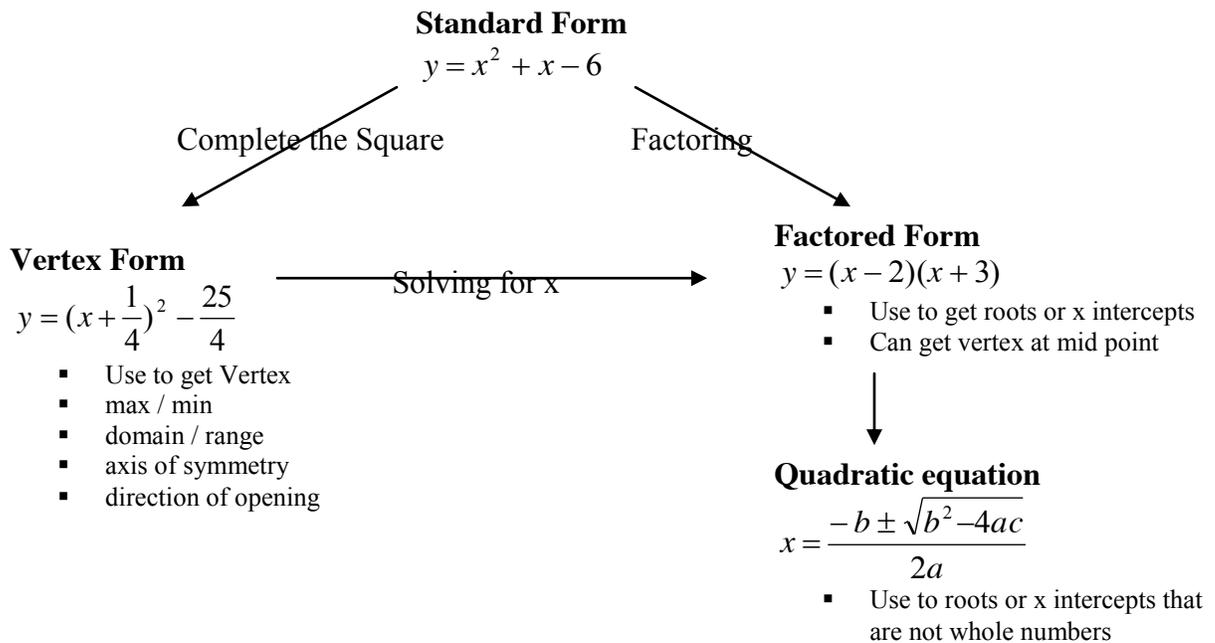
Here's a sample list of skills you must be able to handle (for grade 11 and 12):

- Graphing linear and quadratic equations
- Identifying domain and range in various situations
- Factoring
- Algebraic expansion with squares and cubics
- Dealing with fraction
- Solving 2 equations 2 unknowns using substitutions
- Word problems, especially the various types on quadratics
- And more…

Tip #3: Understanding complex concepts

This is a helpful method when there are a lot of parts to a complex concept, like quadratics in grade 10 and 11. The idea is to see the ``big picture'' and visualize how the parts connect with one another. That way, if the teacher asks a question relating to a specific part of the concept, you'll know how to relate it to the overall picture and find the solution.

Here's an example of our "big picture" for the quadratics unit:

Standard Form
$$y = x^2 + x - 6$$

Complete the Square

Factoring

Vertex Form
$$y = (x + \frac{1}{4})^2 - \frac{25}{4}$$

- Use to get Vertex
- max / min
- domain / range
- axis of symmetry
- direction of opening

Solving for x

Factored Form
$$y = (x - 2)(x + 3)$$

- Use to get roots or x intercepts
- Can get vertex at mid point

Quadratic equation
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Use to roots or x intercepts that are not whole numbers

The teacher has a lot of options when it comes to a unit like quadratics. They might ask you to derive the vertex form from the standard forms (requiring you to complete the square), or to find the roots from the vertex form (requiring you to set y = 0 and solve).

Tip #4: How to tackle TIPS questions

The TIPS section often gives students a headache since the teacher can choose pretty much anything to surprise you. It is extremely difficult to prepare for, but that doesn't mean to give up hope! There are several ways to tackle this problem. One is to try all the difficult questions on your homework or in your textbook on a regular basis. This isn't because your teacher will choose one of these questions for the test, but to train your problem solving skills appropriately.

Another method you might use to prepare yourself for TIPS questions is simplifying the question. Try to come up with a similar question, but make it a little bit easier. After you solve the simplified one, try to determine any similarities between the two questions. You may be able to notice a distinct pattern to the original question, apply the same rules you did with the easier, modified question, and then solve the TIPS problem.

Here's an example of a TIPS question from the grade 12 unit on polynomials:

Determine the quotient of $x^n - y^n \div x - y$

The algebra exponent makes it very hard! But what you could do is create something similar with a number exponent instead: $x^3 - y^3 \div x - y$

You then do the long division based on that. Pay close attention to the exponents. What happens? The exponent is decreased by 1 whenever you divide it! So if you apply that to an algebraic exponent, what happens? You subtract 1 as well. If you apply that idea to the original question, you can deal with the algebraic exponent: n will become n-1, etc.

If you are striving to score in the 80-90% range in grade 11 or 12 math, you must spend the time to train yourself to do well on the TIPS section. You can buy the Waterloo contest booklet from the University of Waterloo (which provides the solutions as well), which usually contains more difficult questions on high school tests. If you can do these, then your grade 11 and 12 questions will be a walk in the park. Try to practice 1 or 2 of these questions a day to prepare yourself.